TAP 225- 1: Radians and angular speed

In many physical situations you are concerned with the motion of objects moving in a circle, such as planets in orbit around the Sun or more mundane examples like wheels turning on a bicycle or washing drying in a spin drier. The measurement of speed can be expressed in several different ways; the following questions are designed to help you become confident in their use.

Rotation

Since a radian is the angle between two radii with an arc length equal to the radius, there are 2π radians in one complete circle.

1. Use a calculator to complete the table of θ in degrees and radians, sin θ, cos θ, and tan θ when θ has values in degrees shown in the table:

θ degree	θ in radians	sin θ	cos θ	tan θ
0.01				
0.1				
0.5				
1				
5				
10				
20				
70				

When θ (in radians) is small what are suitable approximations for:

sin θ, tan θ, cos θ?

The set of questions below consider a simple example of an object moving in a circle at constant speed.

2. Work to two significant figures. Write down the angle in radians if the object moves in one complete circle, and then deduce the number of radians in a right angle.

3. The object rotates at 15 revolutions per minute. Calculate the angular speed in radian per second.

4. A rotating restaurant.

A high tower has a rotating restaurant that moves slowly round in a circle while the diners are eating. The restaurant is designed to give a full 360° view of the sky line in the two hours normally taken by diners.

Calculate the angular speed in radians per second.

5. The diners are sitting at 20 m from the central axis of the tower.

Calculate their speed in metres per second.

Do you think they will be aware of their movement relative to the outside?

Practical Advice

Other examples of physical situations of rotational movement could be usefully discussed. These questions will be of greatest value to students with a weak mathematical background

When θ becomes small and $\sin\theta = \theta$ in radians is used in the angular acceleration discussion. Question 1 helps exemplify this.

Social and Human Context

A number of fairground rides use circular motion and what adds to the 'thrill factor' could be considered here.

Answers and Worked Solutions

1

θ in degrees	θ in radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
0.01	1.75×10^{-4}	1.75×10^{-4}	0.999	1.75×10^{-4}
0.1	1.75×10^{-3}	1.75×10^{-3}	0.999	1.75×10^{-3}
0.5	8.73×10^{-3}	8.73×10^{-3}	0.999	8.73×10^{-3}
1	1.75×10^{-2}	1.75×10^{-2}	0.999	1.75×10^{-2}
5	8.73×10^{-2}	8.72×10^{-2}	0.996	8.72×10^{-2}
10	0.175	0.174	0.985	0.176
20	0.349	0.342	0.940	0.363
70	1.22	0.940	0.342	2.75

It should be clear to students that:-

at small angles $\theta = \sin\theta = \tan\theta$ when θ is in radians

and $\cos\theta = 1$ at small angles.

That the above works quite well even for angles as large as 20 degrees.

2. 2π; $\pi / 2$

3.

$$\text{angular speed} = \frac{15\text{ rev min}^{-1} \times 2\pi}{60\text{ s min}^{-1}} = 1.6 \times 10^{-4}\ \text{rad s}^{-1}$$

4.

$$\text{Angular speed} = \frac{\text{angle in radians}}{\text{time in seconds}} = \frac{2\pi\text{ radians}}{(2 \times 60 \times 60)\text{ seconds}} = 8.7 \times 10^{-4}\ \text{rad s}^{-1}.$$

5.

$v = \omega r$

$v = 8.7 \times 10^{-4}\ \text{s}^{-1} \times 20\,\text{m} = 0.017\ \text{m s}^{-1}$.

They may just be able to perceive it but it is unlikely – they would see the skyline move at less than 2 cm each second.

External References

Question 1 was an adaptation of Revised Nuffield Advanced Physics section D question 8(L)

Questions 2-5 are taken from Advancing Physics Chapter 11, 70W